Helping Students Make Mathematics Connections: Collaborative Visualizations in Smart Classrooms

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Abstract
Researchers have argued that for students to understand mathematics, instruction should provide opportunities for them to make explicit connections between mathematical concepts and encourage them to reflect on these connections. This paper describes a co-designed curriculum activity that combines the use of collaborative visualizations in a technology-enhanced classroom for grades 10 and 11 math students. The aim was to facilitate students’ understanding of links between different mathematical concepts, demonstrated through connections within a set of math problems. Preliminary results showed improved accuracy of students’ math connections between pre- and post-tests, as compared against the teacher’s results. Another important outcome was the curriculum design itself, which successfully engaged students in a collaborative distributed learning activity that leveraged technology and visualizations.

Introduction
There have been ongoing discussions amongst educational researchers concerning how teachers can support students in making connections between mathematics topics (The National Council of Teachers of Mathematics, 2000). Conventional instruction, with its a sequential presentation of materials in textbooks and the rote completion of problem sets, often fails to help students develop a deep understanding. This is particularly true in regard to the interconnections amongst mathematical concepts, which often come across to students as completely separate topics (Hiebert, 1984).

Researchers of mathematics learning have recognized that the task of connecting diverse pieces of information can promote better understanding (Hiebert & Carpenter, 1992). Prior research on concept mapping in mathematics and science education (e.g. Novak, 1998; Hasemann & Mansfield, 1995; Bolte, 1998) has shown that visualizations that display relationships between mathematical ideas can help learners make connections explicit, while encouraging discussions about mathematics. The task of clarifying such relationships through the display of explicit connections can encourage students to jointly negotiate those relationships and solidify their understanding of the underlying mathematics concepts. Particularly when integrated within a technology-enhanced learning environment, this approach can provide opportunities for students to make deep connections among mathematics concepts as well as with their previous knowledge and experiences outside the classroom (Ozel, 2008).
This paper presents a program of research that enables learning by connecting students with peers and richly interactive materials within and between classroom communities and informal settings. Technology-enhanced classrooms can serve as hubs for rich sequences of curriculum activities within the school, but also act as access points through which students can remotely access the knowledge they have constructed from their own homes, or even beyond the scope of their course (Slotta, in press). Carefully developed technology environments can help students form “knowledge communities” via cooperative knowledge construction and their resulting common knowledge base (Bielaczyc, 1999; Scardamalia and Bereiter, 2006; Slotta, in press). Using technology to support students as they create rich and varied relational networks amongst themselves ultimately results in personally relevant curriculum that promotes deep understanding (Woodruff, 2005; Bereiter, 2002).

While the issues and opportunities of technology-enhanced learning are relevant to a very broad array of topics, the domain of mathematics learning is particularly suitable for such investigations. Here, we report on research that addresses the following two research questions: (1) What kinds of learning activities are best suited to integrate students’ home, school and informal learning environments? (2) How can collaborative visualizations allow relevant connections to be formed between math problems and underlying mathematical ideas?

The present study is set within a broader research program that aims to develop a robust smart classroom environment (Slotta, in press; Tissenbaum & Slotta, 2009). In phase 1, the foundations of the technology environment were developed and the initial set of materials developed by a partnership of researchers and a mathematics teacher. Phase 2 consisted of student testing and input, where a select group of volunteer students conducted the technology-enhanced visualization task, forming links between mathematics problems and ideas within the smart classroom environment. Phase 3 involves redesigning the learning environment by working with students and the teacher to ensure that students can effectively use the technology and materials. Phase 4 will include expanded testing with a full class of grade 11 math students creating problems for the system throughout their course. Students will be responsible for uploading these problems and linking ideas, leaving us with a larger set of problems that vary in difficulty and semantic characteristics.

In terms of mathematics learning, the long-term goals of the project are two-fold: (1) to help students develop connections between mathematical concepts; and (2) create a central repository of math problems, solutions and interconnections for students to use either for studying and preparing for extracurricular math competitions.

Method

At the time of submission, phase 2 of the larger scope of the project has been completed and phase 3 is underway. This paper details the design of our smart classroom, the creation of the curriculum, and the outcomes of the initial cohort of students who conducted the collaborative visualization activity. Our
team co-designed a supplemental learning activity to enhance students’ understanding of the mathematics concepts taught in the regular curriculum. The classroom teacher was an active partner in this research.

Participants

Our team included a high-school math teacher, three researchers, and three technology developers, and employed a co-design method (Roschelle, Peneuel, & Shechtmen, 2006). A total of twenty-three student volunteers in grades 10 and 11 from a private, urban high school participated in the pre-test, post-test and curriculum activities.

Materials

A smart classroom with specialized “roomware” (to coordinate the flow of participants and materials amongst various computers, servers, and displays), projectors, and software agents was developed in parallel with the curriculum design process (Figure 1). The curriculum, detailed below, was designed to engage several small groups of students working in parallel as they “tagged” a common set of math problems. In so doing, a collaborative visualization emerged as the curriculum synthesized the combined tags from all groups. A set of thirty problems developed by the teacher belonged to one or more of four category groups: Algebra & Polynomials, Functions & Relations, Trigonometry, and Graphing Functions. The basic goal of this activity was to help students understand the relationships between these four aspects of mathematics by having them visualize the association of math problems with multiple categories. Figure 2 shows an example of two different math problems belonging in the “Functions & Relations” and “Graphing Functions” categories.
Procedure

I. Pre-test

Students were given a paper-based pre-test of ten questions. Rather than solve the questions, they were asked to draw connections between them according to any possible shared mathematics themes. These were collected and compared with the same task as performed by the teacher, who had helped to assemble the problems.

II. Curriculum activity

The points of $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points on the graph of $y = \log(x)$.

Through the midpoint of the line segment $AB$, a horizontal line is drawn to cut the curve at $C(x_3, y_3)$.

Prove that $x^2 = x_1x_2$.

Determine the equation of a parabola having its axis parallel to the $y$-axis and passing through the points (-1,2), (1,-1), and (2,1).

Figure 1. Photo of smart classroom during the curriculum activity

Figure 2. Sample of two mathematics problems that belonged to both “Functions & Relations” and “Graphing Functions” categories
a) Part I - Tagging

Students assembled in the smart classroom, where they were provided with individual laptops and asked to log in to a specially designed system that coordinated the flow of activities for each session. The software agents distributed students into four groups, each specializing in one of the four mathematics categories. Each group gathered in a specified area of the room, where the individual students within that group were presented with the same set of math problems one at a time (on their laptop screens) until the group had exhausted all thirty problems.

For each problem, the student who was viewing it decided whether it should belong in their group’s category, again without solving the problem. A projection screen in front of each group displayed a semantic map of that group’s category, with each of the math problems as nodes. If a problem was labeled (or ‘tagged’) with their group’s category, a connection between the problem and the category was made (Figure 3). This was represented on the screen as a line, linking the problem node to the category node. At the front of the room was a larger screen that showed the collaborative visualization, which displayed an aggregate of the connections made between the questions and all four categories (Figure 4).

Figure 3. Sample of group visualization (created by the “Functions & Relations” group)
b) Part II - Solving

Once the four groups reviewed all the questions, they were provided with pencil, paper, and calculators. The smart classroom software presented students with only those problems that were tagged as belonging to their group’s category as well as at least one other category. Students were then instructed to solve each problem, working as a group. Once completed, they took a photo of their solution using the laptop camera and uploaded it to the system.

c) Part III – Reflecting

Next, the students in each group were shown the tags that other groups had assigned to the questions they just solved and asked to individually vote on the credibility of those tags. For example, students in the “Algebra & Polynomials” group might be presented with a problem that had been tagged by their group as well as the “Trigonometry” group. Upon completing the problem, the students would be informed that the problem had been tagged as being a “Trigonometry” one as well, and asked whether they agreed with the “Trigonometry” connection. After stating their agreement or disagreement, the students would then be prompted to explain their choice in reflection notes.
The group visualizations and the collaborative visualization were updated in real-time – the lines representing connections that had stronger consensus became thicker, and uploaded solutions appeared as new nodes.

IV. Post-test
Students participated in an online survey where they identified ten math problems as “Algebra & Polynomials”, “Functions & Relations”, “Trigonometry”, and/or “Graphing Functions”. They could check off more than one category, and were also asked to point out any other themes the questions were related to. Boxes for comments were made available for students’ input on their experience on the pre-test, curriculum activity and post-test.

Findings and Discussion
I. Data Analysis
At the time of submission, our assessment data consist of a pre- and post-test, as well as evaluation of the collaborative visualization from Part I of the curriculum design. Accuracy scores were compiled by looking at the group average of correct connections against the total number of connections made by the teacher. Following previous work using mathematics concept maps (Hasemann & Mansfield, 1995), the “structuredness” for each set of data was evaluated with the number of connections made compared against the total number of potential connections.

II. Accuracy & Structuredness
On the pre-test, students made very few connections between problems, on average, compared with their teacher (Figure 5), with an accuracy rate of 20.00%. The structuredness level was 18.89%. During the curriculum activity, students made substantially more connections between problems and categories, with a structuredness rate of 45.24% and a much greater accuracy rate of 53.31%. On the post-test, the accuracy rate for the smart classroom group was 63.75% (Figure 6), and the structuredness was 38.50%.

These preliminary findings, while representing only a small number of participants, showed an upward trend of increasing accuracy and structuredness for the experimental condition. The improved accuracy from the pre-test to the curriculum activity and post-test suggests the importance of how we ask students to make connections to problems, with greater accuracy derived from a collaborative design which shares responsibility. The structuredness, which measured students’ recognition of the connections, shows increasing willingness to characterize math problems from different perspectives. This is important for the latter phases of the project, where students will be uploading and interconnecting their own math problems.
III. Student comments

Overall, students found visualization useful in showing different mathematical themes from which a problem could be approached. One student
indicated that the visualization was helpful when he could not solve a problem. Students also stated that, over time and with more contributors, the system would become increasingly valuable for studying purposes.

Students also commented that they became more cognizant of the connections amongst mathematics ideas and themes. It is noteworthy that students gained awareness that one could discuss properties of math problems and their relevant themes rather than simply answer them.

IV. Discussion

The collaborative visualizations used in this study did more than help students make connections between math problems and themes. They were also a means of assessing the connections that students made, by comparing their answers to those of the teacher (or another normative source). The collaborative visualizations also provided a record of the aggregated connections, artifacts (e.g. problems, solutions) and communications amongst students over time, which can be used to inform the design of subsequent learning activities.

The aim of our curriculum activity was to increase the depth of students’ mathematics understanding by breaking down learning goals into manageable activities, supported by a technology-enhanced learning environment. The preliminary findings are encouraging. Moving forward through the larger project, our co-design team will make changes to the smart classroom environment and curriculum materials, taking into account student input and feedback gathered in this study. We will also begin applying our basic approach to collaborative visualizations in other disciplines, such as Science, English and the Arts.
References


